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BRIEF COMMUNICATION

THE EFFECT OF TEMPERATURE DEPENDENT PROPERTIES ON THE REWETTING VELOCITY

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Although there are numerous works on rewetting of hot surfaces, the effect of temperature dependent properties has gained little attention. A one-dimensional analysis is proposed here, based on the heat conduction in the solid. The temperature dependent properties are written in the form: $A_i = A_{i,0}\,[1 + \gamma_i\,(T - T_r)]$, $i = 1, 2, 3, 4$, where A_i represents either the density ρ, thermal conductivity k, specific heat c, or the thickness of the slab δ; $A_{i,0}$ is the value of the property at the reference temperature T_r and γ_i are various constants.

We adopt the one-dimensional model presented by Séméria & Martinet (1965) and later by Yamanouchi (1968), which is valid for small Biot and Peclet numbers only.

The heat conduction equation for a frame of reference moving with the quench front ($x = 0$) at the rewetting velocity u is

$$\frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] + \rho(T)c(T)u\frac{\partial T}{\partial x} - \frac{h}{\delta(T)}(T - T_s) = 0, \qquad [1]$$

where the heat transfer coefficient h is assumed to be constant in the wetted region ($x \le 0$) and zero in the dry region ($x > 0$). The boundary conditions are:

$$x \to -\infty \qquad T \to T_s, \qquad [2]$$

$$x \to \infty \qquad T \to T_w, \qquad [3]$$

$$x = 0 \qquad T\text{: smoothly continuous.} \qquad [4]$$

The axial coordinate is denoted by x, T_s and T_w are the coolant and initial wall temperatures, respectively, and the temperature dependent properties are:

$$k = k_0[1 + \gamma_1(T - T_r)], \quad c = c_0[1 + \gamma_2(T - T_r)],$$
$$\rho = \rho_0[1 - \gamma_3(T - T_r)], \quad \delta = \delta_0[1 + \gamma_4(T - T_r)].$$

We define the following nondimensional variables:

$$\theta \equiv (T - T_r)/(T_w - T_r), \quad \eta \equiv x/\delta_0,$$
$$Pe \equiv \rho_0 c_0 \delta_0 u/k_0, \quad Bi \equiv h\delta_0/k_0,$$

$\epsilon_i \equiv \gamma_i(T_w - T_r)$, $i = 1, 2, 3, 4$, where the analysis is restricted to values of T_w such that $\epsilon_i \ll 1$.

The properties will resume the following form:

$$k = k_0(1 + \epsilon_1\theta), \quad c = c_0(1 + \epsilon_2\theta),$$
$$\rho = \rho_0(1 - \epsilon_3\theta), \quad \delta = \delta_0(1 + \epsilon_4\theta)$$

and the nondimensional formulation of [1]–[4] becomes:

$$\frac{\partial}{\partial\eta}\left[(1+\epsilon_1\theta)\frac{\partial\theta}{\partial\eta}\right] + Pe(1+\epsilon_2\theta)(1-\epsilon_3\theta)\frac{\partial\theta}{\partial\eta} - Bi\frac{\theta-\theta_s}{1+\epsilon_4\theta} = 0 \qquad [5]$$

$$\eta \longrightarrow -\infty \qquad \theta \longrightarrow \theta_s, \qquad [6]$$

$$\eta \longrightarrow \infty \qquad \theta \longrightarrow 1, \qquad [7]$$

$$\eta = 0 \qquad \theta\text{: smoothly continuous.} \qquad [8]$$

Let us introduce the following formal asymptotic expansion for θ in terms of the four parameters ϵ_1, ϵ_2, ϵ_3, and ϵ_4

$$\theta(\eta) = \theta_0 + \epsilon_1\theta_1 + \epsilon_2\theta_2 + \epsilon_3\theta_3 + \epsilon_4\theta_4 + \text{H.O.T.} \qquad [9]$$

Substituting [9] into [5] and equating identical powers of ϵ_1, ϵ_2, ϵ_3 and ϵ_4, result in the following set of equations for θ_0, θ_1, θ_2, θ_3 and θ_4 with the appropriate boundary conditions, representing the mathematical formulation for the wetted region only ($\eta \le 0$):

$$\theta_0'' + Pe\theta_0' - Bi(\theta_0 - \theta_s) = 0; \qquad \theta_0(-\infty) = \theta_s \quad \theta_0(0) = \theta_q, \qquad [10]$$

$$\theta_1'' + Pe\theta_1' - Bi\theta_1 + \theta_0\theta_0'' + \theta_0'^2 = 0; \qquad \theta_1(-\infty) = 0 \quad \theta_1(0) = 0, \qquad [11]$$

$$\theta_2'' + Pe\theta_2' - Bi\theta_2 + Pe\theta_0\theta_0' = 0; \qquad \theta_2(-\infty) = 0 \quad \theta_2(0) = 0, \qquad [12]$$

$$\theta_3'' + Pe\theta_3' - Bi\theta_3 - Pe\theta_0\theta_0' = 0: \qquad \theta_3(-\infty) = 0 \quad \theta_3(0) = 0, \qquad [13]$$

$$\theta_4'' + Pe\theta_4' - Bi\theta_4 + \theta_0\theta_0'' + Pe\theta_0\theta_0' = 0; \quad \theta_4(-\infty) = 0 \quad \theta_4(0) = 0, \qquad [14]$$

where primes denote differentiation with respect to η and $\theta_q = (T_q - T_r)/(T_w - T_r)$ is the nondimensional rewetting temperature. The solutions to [10]–[14] are

$$\theta = \theta_s + (\theta_q - \theta_s)e^{\alpha\eta} \qquad [15]$$

$$\theta_1 = \frac{2\alpha}{2\alpha-\beta}(\theta_q-\theta_s)^2(e^{\alpha\eta}-e^{2\alpha\eta}) - \frac{\alpha^2}{\alpha-\beta}\theta_s(\theta_q-\theta_s)\eta e^{\alpha\eta}, \qquad [16]$$

$$\theta_2 = \frac{Pe}{2\alpha-\beta}(\theta_q-\theta_s)^2(e^{\alpha\eta}-e^{2\alpha\eta}) - \frac{\alpha Pe}{\alpha-\beta}\theta_s(\theta_q-\theta_s)\eta e^{\alpha\eta}, \qquad [17]$$

$$\theta_3 = -\theta_2, \qquad [18]$$

$$\theta_4 = \frac{\alpha+Pe}{2\alpha-\beta}(\theta_q-\theta_s)^2(e^{\alpha\eta}-e^{2\alpha\eta}) - \frac{\alpha(\alpha+Pe)}{\alpha-\beta}\theta_s(\theta_q-\theta_s)\eta e^{\alpha\eta}, \qquad [19]$$

where

$$\alpha = -\frac{Pe}{2} + \left(\frac{Pe^2}{4} + Bi\right)^{1/2}; \quad \beta = -\frac{Pe}{2} - \left(\frac{Pe^2}{4} + Bi\right)^{1/2}$$

Following Duffey & Porthouse (1973), we use a heat balance approach: the heat flowing across the surface to the liquid per unit time is equal to the enthalpy change in the slab per unit time, at the quasi-steady state considered here. This leads to

$$\int_{-\infty}^{0} hl(T - T_s)\,dx = \int_{-\infty}^{\infty} \rho cl\delta u\frac{\partial T}{\partial x}\,dx, \qquad [20]$$

where l is the breadth of the slab. After dividing both sides by l and substituting the

Table 1. Material properties used in the calculations, Yu *et al.* (1977) (T is in °C)

Material	Thermal conductivity ($Wm^{-1}\,°C^{-1}$)	Specific heat ($J\,kg^{-1}\,°C^{-1}$)	Density ($kg\,m^{-3}$)
Inconel	$14.0(1 + 1.200 \times 10^{-3}\,T)$	$488(1 + 4.467 \times 10^{-4}\,T)$	$8420(1 - 4.311 \times 10^{-5}\,T)$
Zircaloy	$10.0(1 + 1.610 \times 10^{-3}\,T)$	$285(1 + 3.509 \times 10^{-4}\,T)$	$6573(1 - 1.161 \times 10^{-5}\,T)$

nondimensional variables, [20] reduces to

$$\int_{-\infty}^{0} Bi(\theta - \theta_s)\mathrm{d}\eta = \int_{-\infty}^{\infty} Pe(1 + \epsilon_2\theta)(1 - \epsilon_3\theta)(1 + \epsilon_4\theta)\frac{\partial\theta}{\partial\eta}\mathrm{d}\eta. \qquad [21]$$

After substituting into [21] the proper expressions for the temperature and properties in terms of ϵ_1, ϵ_2, ϵ_3 and ϵ_4 and carrying out the integrations, the following implicit expression for the nondimensional rewetting velocity Pe is obtained:

$$\begin{aligned} &Bi\left\{\frac{1}{\alpha}(\theta_q - \theta_s) + \frac{1}{2\alpha(2\alpha - \beta)}(\theta_q - \theta_s)^2[2\alpha\epsilon_1 + Pe(\epsilon_2 - \epsilon_3) + (\alpha + Pe)\epsilon_4]\right. \\ &\left. + \frac{1}{\alpha(\alpha - \beta)}\theta_s(\theta_q - \theta_s)[\alpha\epsilon_1 + Pe(\epsilon_2 - \epsilon_3) + (\alpha + Pe)\epsilon_4]\right\} \\ &= Pe(1 - \theta_s)\left[1 + \frac{\epsilon_2 - \epsilon_3 + \epsilon_4}{2}(1 + \theta_s)\right]. \end{aligned} \qquad [22]$$

Inspection of [22] reveals that when $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 0$ it reduces, after simple manipulations, to the well known relation for fixed properties, e.g. Elias & Yadigaroglu (1978):

$$\frac{Bi^{1/2}}{Pe} = \left[\frac{1 - \theta_s}{\theta_q - \theta_s}\left(\frac{1 - \theta_s}{\theta_q - \theta_s} - 1\right)\right]^{1/2}. \qquad [23]$$

Furthermore, table 1 shows that ϵ_3 is smaller than ϵ_2 by one order of magnitude, therefore the density dependence on temperature may be neglected in [22]. As the linear thermal

Table 2. Values of rewetting velocities for fixed and variable properties in mm/s for $T_q = 260$ °C, and the relative error E (in percents)

		Inconel			Zircaloy		
T_w °C	T_s °C	20	100	200	20	100	200
	u_F	1.54	1.18	0.57	2.58	1.97	0.96
350	u_V	1.53	1.17	0.57	2.53	1.94	0.95
	E	0.45	0.22	0.21	1.84	1.45	0.68
	u_F	1.16	0.87	0.40	1.96	1.47	0.67
400	u_V	1.13	0.85	0.39	1.87	1.41	0.65
	E	2.48	2.15	1.60	4.81	4.25	3.27
	u_F	0.94	0.70	0.31	1.61	1.19	0.53
450	u_V	0.90	0.67	0.30	1.49	1.11	0.50
	E	4.37	3.97	3.33	7.55	6.85	5.73
	u_F	0.80	0.58	0.25	1.37	1.00	0.43
500	u_V	0.75	0.55	0.24	1.25	0.92	0.40
	E	6.16	5.69	4.99	10.11	9.30	8.07
	u_F	0.70	0.50	0.21	1.17	0.85	0.36
550	u_V	0.64	0.47	0.20	1.07	0.78	0.34
	E	7.87	7.35	6.60	9.53	8.64	7.36
	u_F	0.62	0.44	0.19	1.05	0.75	0.32
600	u_V	0.56	0.41	0.17	0.94	0.68	0.29
	E	9.51	8.95	8.15	11.92	10.95	9.59

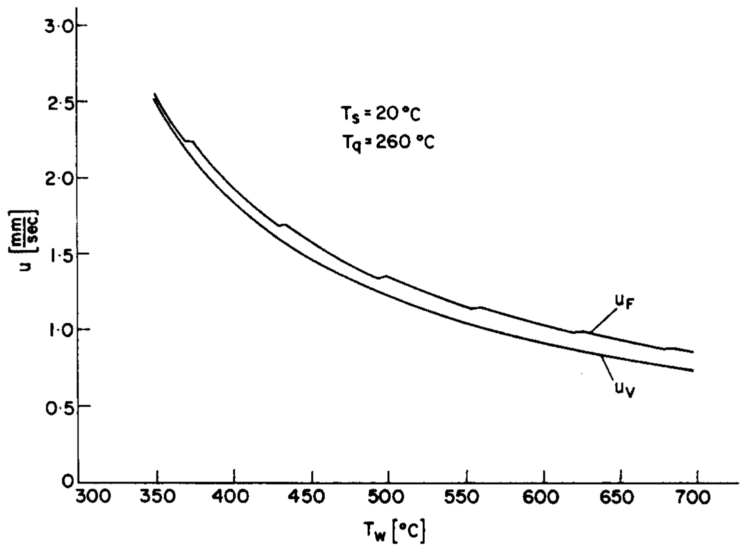

Figure 1. Rewetting velocity vs Zircaloy wall temperature for $T_s = 20°$.

expansion coefficient for metals has an order of magnitude 10^{-5} per °C, the expansion of the slab (ϵ_4) may also be neglected in [22].

The properties of two different materials are used to demonstrate the effect of variable properties on the rewetting velocity, cf. table 1. We take for the slab thickness a typical value of 1 mm and values for the heat transfer coefficient such that $Bi = 0.1$. The rewetting velocity for variable properties u_V was calculated numerically from [22], whereas the rewetting velocity for fixed properties u_F was obtained explicitly from [23]. Table 2 shows values of u_F and u_V and the relative error E, of u_F with respect to u_V, for various wall and coolant inlet temperatures and for $T_q = 260$ °C, a value widely used by investigators, e.g. Elias & Yadigaroglu (1978), table 1.

u_F was evaluated at the initial wall temperature T_w since the wall–coolant interface temperature is close to T_w when, as for water, $\sqrt{\rho c k}_{\text{wall}} \gg \sqrt{\rho c k}_{\text{coolant}}$, e.g. Gunnerson & Yackle (1981). Figures 1 and 2 show a comparison of the rewetting velocities u_F and u_V as a function of the initial wall temperature for various water inlet temperatures when the metal is Zircaloy. The nonmonotonous behavior of u_F in these figures deserves special discussion.

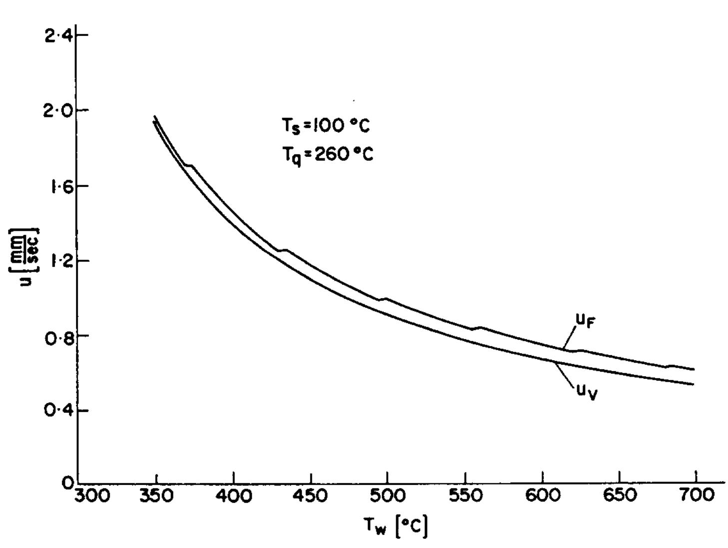

Figure 2. Rewetting velocity vs Zircaloy wall temperature for $T_s = 100$ °C.

Models neglecting temperature dependence of the properties evaluate them at a single arbitrary temperature within the range of interest. Thus, in the well known expression for the rewetting velocity

$$u = \frac{1}{\rho c}\left(\frac{hk}{\delta}\right)^{1/2} \frac{T_q - T_s}{(T_w - T_s)^{1/2}(T_w - T_q)^{1/2}}, \qquad [24]$$

u is a monotonous function of T_w as ρ, c, k and δ are constant. When comparing u_F and u_V, an evaluation of the properties for u_F at a single arbitrary temperature may lead to erroneous conclusions. This is due to a growing artificial discrepancy between u_F and u_V at values of T_w much different from that single temperature.

Therefore the following relation for u_F is used instead of [24], evaluating the properties at the relevant initial wall temperature T_w (as mentioned above):

$$u_F = \frac{1}{\rho_0 c_0}\left(\frac{hk_0}{\delta_0}\right)^{1/2} \cdot \frac{[1 + \gamma_1(T_w - T_r)]^{1/2}(T_q - T_s)}{[1 - \gamma_3(T_w - T_r)][1 + \gamma_2(T_w - T_r)][1 + \gamma_4(T_w - T_r)^{1/2}(T_w - T_s)^{1/2}(T_w - T_q)^{1/2}}. \qquad [25]$$

It is obvious that the last expression is not a monotonous function of T_w and has points of maxima.

Table 2 and figures 1 and 2 indicate the following trends: the effect of variable properties increases with increasing high values of initial wall temperature and with decreasing values of fluid inlet temperature: the effect is stronger for Zircaloy than for Inconel.

It may be concluded that for present models the effect of variable properties can be neglected, as uncertainties in the rewetting temperature and heat transfer coefficient are still very large. As these uncertainties are resolved, variable properties should be accounted for in future models for certain relevant parameters.

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REFERENCES

DUFFEY, R. B. & PORTHOUSE, D. T. C. 1973 The Physics of Rewetting in Water Reactor Emergency Core-Cooling. *Nucl. Eng. Des.* **25,** 379–394.

ELIAS, E. & YADIGAROGLU, G. 1978 The Reflooding Phase of LOCA in PWRs Part II. Rewetting and Liquid Entrainment. *Nuclear Safety* **19,** 160–175.

GUNNERSON, F. S. & YACKLE, T. R. 1981 Quenching and Rewetting of Nuclear Fuel Rods. *Nucl. Technol.* **54,** 113–117.

SÉMÉRIA, R. & MARTINET, B. 1965 Calefaction Spots on a Heating Wall: Temperature Distribution and Resorption. Proc. Inst. Mech. Eng. **180,** 192–205.

YAMANOUCHI, A. 1968 Effect of Core Spray Cooling in Transient States after Loss of Coolant Accident. *J. Nucl. Sci. Technol.* **5,** 547–558.

YU, S. K. W., FARMER, P. R., & CONEY, M. W. E. 1977 Methods and Correlations for the Prediction of Quenching Rates on Hot Surfaces. *Int. J. Multiphase Flow* **3,** 415–443.